A Water Supply Optimization Problem for Plant Growth Based on GreenLab Model

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ABSTRACT. GreenLab is a structural-functional model for plant growth based on multidisciplinary knowledge. Its mathematical formalism allows dynamic simulation of plant growth and model analysis. A simplified soil water balance equation is introduced to illustrate the interactions and feedbacks between the plant functioning and water resources. A water supply optimization problem is then described and solved: the sunflower fruit weight is optimized with respect to different water supply strategies in a theoretical case. Genetic algorithms are used to solve this mixed integer nonlinear problem. The optimization results are analyzed and reveal possible agronomic applications.

RÉSUMÉ. GreenLab est un modèle structure-fonction de croissance des plantes. Son formalisme mathématique permet la simulation dynamique de la croissance et l'analyse du modèle. Dans cet article est introduit une équation bilan de l'eau dans le sol afin de décrire les interactions entre la croissance de la plante et les ressources en eau disponibles. Un problème d'optimisation des apports d'eau au cours de la croissance est présenté et résolu par les algorithmes génétiques : le poids du fruit de tournesol est maximisé en fonction de différentes stratégies d'apports d'eau, pour une quantité d'eau totale identique. Le formalisme présenté est intéressant en ce qu'il ouvre la voie à d'importantes applications en agronomie.

KEYWORDS : modèle de croissance de plantes, resources en eau, algoritmes génétiques, problème d'optimisation mixte

MOTS-CLÉS : plant growth model, soil water balance, genetic algorithm, mixed integer nonlinear programming problem

1. Introduction

Plant growth models play an essential role in agronomy, botany and computer graphics. Different kinds of models, such as process-based models [5], geometric models [3] or functional-structural models [7], have been developed for special applications and they usually remain valid only for these. GreenLab model, see [2], tries to present a more general mathematical description of plant growth combining both physiology and architecture. It is based on multidisciplinary knowledge, including botany, eco-physiology, agronomy, mathematics and computer sciences. The development of GreenLab is a constant process of balancing the simplicity and complexity when choosing and adapting the biological and mathematical knowledge to form an efficient dynamical plant model useful for a wide variety of applications in agronomy and forestry [6].

In this paper, a simplified soil water balance equation is introduced to GreenLab model in order to take into account the interactions and feedbacks between the plant functioning and water resources. A water supply optimization problem is then formularized and solved with genetic algorithms, showing possible applications of GreenLab in optimization and control for agronomy.

2. GreenLab model

Like its predecessor AMAPhydro [1], GreenLab describes plant architecture at organ level. The evolution of the plant structure called organogenesis is periodical and we define the growth cycle (GC) as the thermal time necessary for each plant axis to develop a new growth unit (GU). The duration of GC can vary from several days (cottons) to one year (temperate trees), but the sum of the daily temperatures needed to create a new GU of each GC is quite constant.

At every GC, the plant produces biomass by leaf photosynthesis. If we consider that every leaf undergoes the same microclimatic conditions, $Q(n)$ the biomass produced by the photosynthesis of all the leaves during GC n can be formularized as an empirical nonlinear function F of the environmental conditions $E(n)$, the number of leaves, their surface areas, and some hidden parameters $r1, r2 \ldots$ In GreenLab [1], we choose:

$$Q(n) = F(S, E(n), r1, r2) = \sum_{i=1}^{n} \left(N_l(i,n) \cdot \frac{E(n)}{\frac{r_1}{S(i,n)} + r_2} \right) \qquad [1]$$

where $N_l(i,n)$ is the number of leaves n of chronological age (CA) i, at GC n, (these leaves appeared at GC $n - i + 1$, and $S(i,n)$ is their surface area. If the leaf thickness is constant, its surface area will be proportional to the biomass accumulated by the leaf.

$r1, r2$ are parameters of F to assess. $E(n)$ is the average biomass production potential depending on environmental factors, such as light, temperature and soil water content.

The biomass produced by photosynthesis is redistributed among all the organs according to their demands d_o:

$$d_o(j) = P_o \phi_o(j) \qquad [2]$$

which depends on organ CA j and organ type o ($o = a, e, c, f, m$ and refers respectively to leaf, internode, layer, female flower and male flower). P_o are the organ sink strengths and are model hidden parameters. ϕ_o are normalized distribution functions characterizing the evolution of the sink strengths from CA 1 to CA t_o , t_o being the organ lifespan. Thus, the total biomass demand of the plant at GC n is:

$$D(n) = \sum_o \sum_{a=1}^{t_o} N_o(a,n) \cdot d_o(a) \qquad [3]$$

where $N_o(a,n)$ is the number of o-type organs of CA a at GC n. This instantaneously leads to the calculation of the biomass increment $\Delta q_o(a,n)$ and total cumulated biomass $q_o(a,n)$ of any o-type organ of CA a at current GC n:

$$\begin{array}{l} \Delta q_o(a,n) = \frac{d_o(a,n)}{D(n)} \cdot Q(n-1) \\ q_o(a,n) = \sum\limits_{j=1}^{a} \Delta q_o(j, n-(a-j)) = P_o \sum\limits_{j=1}^{a} \frac{d(j) \cdot Q(n-(a-j)-1)}{D(n-(a-j))} \end{array} \qquad [4]$$

3. Mathematical description of the water supply problem

3.1. Plant growth interacting with water resources in soil

Plants participate to soil water circulation by transpiration. Water is taken from soil by roots and flows through the plant hydraulic network up to the leaves, where water is transpired to provide necessary energy fluxes for photosynthesis. The water content in the superior soil layers, named soil moisture, is important for the study of bio-geophysical processes in agricultural or forestry ecosystems. Soil water balance is achieved when we simplify this complex soil-plant system by concentrating on plant transpiration, soil evapotranspiration, and water supply from both irrigation and precipitations.

Suppose $Q_u(t)$ is the water content in soil per surface unit. It can be considered as a potential. The loss of water by evapotranspiration is:

$$dQ_u(t) = -c_1 \cdot (Q_u(t) - Q_{umn}) \cdot dt \qquad [5]$$

where Q_{umn} corresponds to the wilt point of soil water content beneath which the plant cannot extract water from soil. c_1 is an evapotranspiration coefficient. Likewise, if $U(t)$ is the water supply at t, the water gained by the soil is

$$dQ_u(t) = c_2(Q_{umx} - Q_u(t))U(t) \cdot dt \quad [6]$$

where Q_{umx} is the water field capacity above which the water flows away and c_2 is an absorption coefficient.

Thus the differential equation for the evolution of the soil water content is:

$$dQ_u(t) = (-c_1(Q_u(t) - Q_{umn}) + c_2(Q_{umx} - Q_u(t)) \cdot U(t))dt \quad [7]$$

Figure 1 shows the fitting results of the calibrated soil moisture model (7) using measurements of the soil water content and rainfalls done in Ivory Coast.

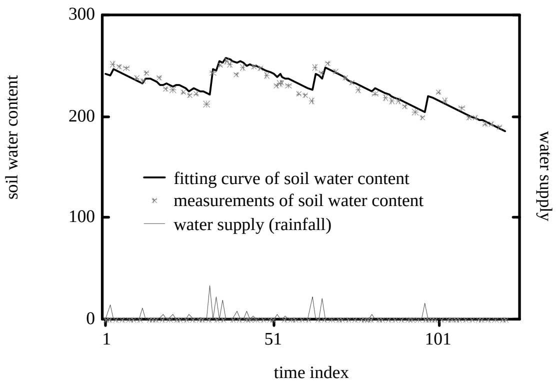

Figure 1. *Fitting results of the calibrated soil moisture model*

Considering plant transpiration, Equation (7) becomes:

$$\frac{dQ_u(t)}{dt} = \underbrace{-c_1(Q_u(t) - Q_{umn})}_{\text{soil evapotranspiration}} + \underbrace{c_2(Q_{umx} - Q_u(t)) \cdot U(t)}_{\text{water absorption}} - \underbrace{PT(t)}_{\text{plant transpiration}} \quad [8]$$

where $PT(t)$ is the plant transpiration and is linearly proportional to plant biomass production calculated by equation (1). The discretized form of Equation (8) at GC scale is:

$$Q_u(n+1) = (1 - c_1 - c_2 \cdot U(n))Q_u(n) + Q_{umn}c_1 + Q_{umx}c_2 \cdot U(n) - \rho \cdot Q(n) \quad [9]$$

where $Q_w(n)$ is the soil water content at GC n, $U(n)$ is the water supply during GC n, $Q(n)$ is the plant biomass production during GC n and ρ is the ratio between plant transpiration and plant biomass production.

Without stress regarding light and temperature conditions, the $E(n)$ biomass production potential E is linearly proportional to the current soil water content:

$$E(n) = K \cdot \frac{Q_w(n) - Q_{wmn}}{Q_{wmx} - Q_{wmn}} \qquad [10]$$

Equations (1), (4), (9) and (10) provide the whole mathematical formalism of the soil-plant system that enables us to study the interactions between plant and water resources in soil. Note that all organ morphologic characteristics can be calculated by (4) and some additional allometric rules. They can be considered as observations of the soil-plant system. Once the observations are measured, hidden model parameters can be tracked back by inverse methods. In [2], it was shown that E the calibration of some hidden parameters, such as sink strengths P_o and hydraulic resistances r_1, r_2 remain stable. Therefore these parameters can be considered as internal endogenous factors. In [8], Zhan calibrates these parameters for several cultivated plants, such as maize, cotton, sunflower and tomato. In this paper, we choose a 63 GC sunflower for the water supply optimization problem. We use the calibration values for internal endogenous parameters, and the external environmental parameters, such as c_1, c_2, etc., are set to empirical values from previous studies.

3.2. Formulation of plant water supply optimization problem

With unlimited water supply, the plant growth is optimal. Nevertheless, in numerous cases, water reserves are limited. For a given quantity of water supply, the fruit yield will depend on the irrigation strategy during the plant growth.

In order to alleviate the calculation load, instead of optimizing the water supply at each GC, we try to find an optimal water distribution curve of the total water supply amount among a cluster of curves. We use parameterized beta function to generate a cluster of water supply curves. Given beta function parameters a and b, and the total water supply amount WT, the water supply at GC i, $i \in [1, N]$, noted U_i, is:

$$U_i = \frac{\text{WT}}{S} \left(\frac{i - 0.5}{N}\right)^{a-1} \left(1 - \frac{i - 0.5}{N}\right)^{b-1}$$

where S is a normalization factor of beta function:

$$S = \sum_{i=1}^{N} \left(\frac{i - 0.5}{N}\right)^{a-1} \left(1 - \frac{i - 0.5}{N}\right)^{b-1}$$

We introduce two variables *date*, the GC at which the water supply starts, and *freq*, the water supply frequency, to simulate a practical water supply strategy. The water supply is redistributed at GC i, with $i = j \times$ freq $+$ date, $j \in \mathbb{N}$ and $1 \leq i \leq N$. For the other GC, water supply is set to zero. The water supply optimization problem is then formularized as a mixed integer nonlinear programming problem P1 (MINLP)

$$\begin{array}{ll} \max_{x \in \mathbb{R}^4} f(x) & \\ \text{subject to,} & 1 \leq x_1, x_2 \leq 100 \\ & 1 \leq x_3, x_4 \leq N \end{array}$$

where x_1, x_2 are the continuous beta law variables a, b and x_3, x_4 are the integer variables *date* and *freq*. The bounds for x_1, x_2 ensures a sufficient amount of distribution curves. N is the sunflower total number of GC.

4. Solution of MINLP problem P1 using genetic algorithms

Genetic algorithms belong to a category of stochastic search techniques for optimization. A population of elements in the search space evolves generation after generation towards a better fitness, according to genetic-like rules. The GA used in this paper is a standard one [4]. The continuous parameters a, b, and integer parameters *date, freq*, are encoded as arrays of binary bits, named chromosomes. The fitness function is the final fruit weight obtained with the corresponding water supply strategy. To generate a new population, we first select chromosomes with a probability proportional to their fitness. Then, the pairwise selected chromosomes exchange parts of their chromosomes with a crossover probability *Pc*. Finally, mutation is carried out by flitting bits randomly with a mutation probability *Pm*.

To solve the water supply optimization problem, we take a population size of 35, *Pm=0.05, Pc=0.4* and we stop the algorithm after 150 generations. The best chromosome is always kept for the next generation. The MINLP genetic algorithm solver was tried 3 times with randomly generated initial populations. The optimal parameter values, optimal fruit weight and the generation at which the best chromosome appears are listed in Table 1. The optimal parameter values are stable for the 3 tries.

Solution Number	Appearing Generation	Optimal parameter values				Optimal fruit weight (g)
		a	b	date	freq	
1	110	1.3686	1.1934	1	2	1196.3
2	92	1.3263	1.1632	1	2	1196.6
3	108	1.3263	1.1571	1	2	1196.6

Table 1. *Optimization results of MINLP GA solver*

Different water supply strategies and their fruit accumulation are compared in Figure 2. (a) is a linear water supply strategy, with *freq = 1, date = 1*; (b) is the optimal water supply strategy with *freq = 1, date = 1*; (c) is the optimal results of the problem P1 with arbitrary *freq* and *date* values. 3D plant geometries of each strategy are also calculated and compared. The fruit weight (J) and sunflower height (H) are quite different for water distribution strategies (a), (b), (c). With the optimal strategy (c), the fruit is 18% heavier than with strategy (b) and the plant is 15% higher. It is interesting to note that with strategy (b), the fruit is 51% heavier than with strategy (a), but the plant is smaller. It is due to the abundance of water supply at early GC, favoring internode growth, and deficient water supply after the fruit appearance for strategy (a).

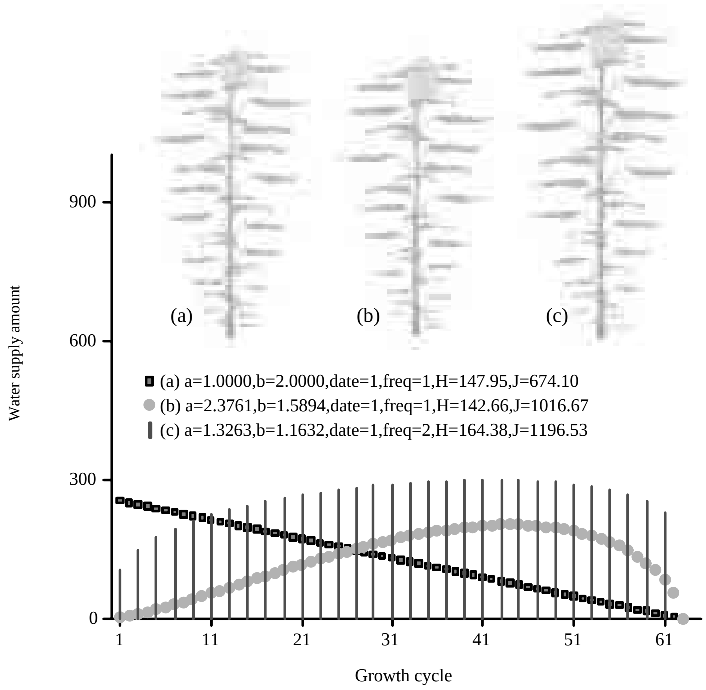

Figure 2. *Comparison of different water supply strategies and the resulting fruit weight*

5. Conclusion

In this paper, we have extended GreenLab model in order to take into account available water resources in soil. It has been done by deriving a simplified water balance equation.

The mathematical formalism introduced and the plant growth description by a dynamical system has allowed us to define a water supply optimization problem. It has been solved using genetic algorithms and the numerical results obtained in a theoretical case give the best water supply strategy in order to obtain a maximum fruit weight. The results are rather preliminary, since external parameters are set empirically, and their calibration and validation are needed. However, the problem solved is a very good example of the kind of applications in agronomy or forestry that we want to achieve. We are confident that the mathematical formalism introduced in the functional-structural model GreenLab should lead to this goal.

6. References

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